A Proof of Eq.(18)

Equation (18) is

$$9L_n = 3L_{n-1} + 6M_{n-1}. \tag{1}$$

This can be derived by the straightforward calculation using the inverse of L. But there is more easy method which uses the solution of

$$\begin{pmatrix} \Phi \\ \Phi \\ \Phi \end{pmatrix} = \begin{pmatrix} L & M & M \\ M & L & M \\ M & M & L \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}. \tag{2}$$

The solution of the above simultaneous equations is

$$\begin{pmatrix} I \\ I \\ I \end{pmatrix}, \tag{3}$$

where $I = \Phi/(L + M + M)$. Therefore, the summation of Eq.(2) is

$$3\Phi = (3L + 6M)I. \tag{4}$$

Using the above relation,

$$\Phi = \frac{3L + 6M}{9} 3I = \frac{3L + 6M}{9} \sum_{i=1,3} I_i \tag{5}$$

is obtained. Since $3I$ is the current of the upper layer, the coefficient $(3L + 6M)/9$ is the inductance of the upper layer. Hence, Eq. (18) is satisfied.